# INSTITUTE FOR MATHEMATICAL RESEARCH

## OLIMPIAD MATEMATIK UNIVERSITI MALAYSIA 2022
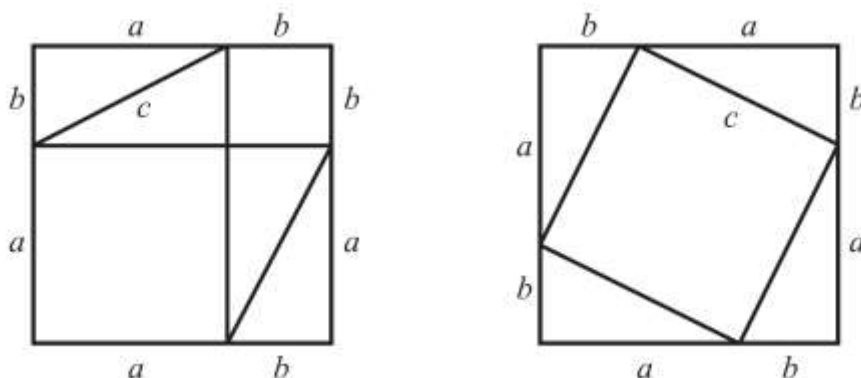## OMUM2022

### PEPERIKSAAN PERINGKAT SARINGAN

Tarikh : 24 September 2022

Masa : 9.30 am – 12.30 pm

Tempoh : 3 jam

Arahan kepada calon:

1. Jawab **SEMUA** soalan.

2. Kalkulator adalah **TIDAK** dibenarkan sepanjang peperiksaan berlangsung.

3. Soalan adalah dalam bahasa Inggeris.

4. Markah diberi untuk jalan kerja dan jawapan yang tepat.

## Question 1

Given two squares below.



Explain how these squares visualize the proof of Pythagoras Theorem.

[10 marks]

## Question 2

Show that $\sqrt{19} + \sqrt{99} < \sqrt{20} + \sqrt{98}$.

[10 marks]

## Question 3

Find all continuous functions $f: \mathbb{R} \to \mathbb{R}$ such that $f(2022x) - f(2021x) = 674x$.

[10 marks]

## Question 4

Prove that

$$\frac{1}{1^4 + 1^2 + 1} + \frac{2}{2^4 + 2^2 + 1} + \cdots + \frac{2022}{2022^4 + 2022^2 + 1} < \frac{1}{2}.$$

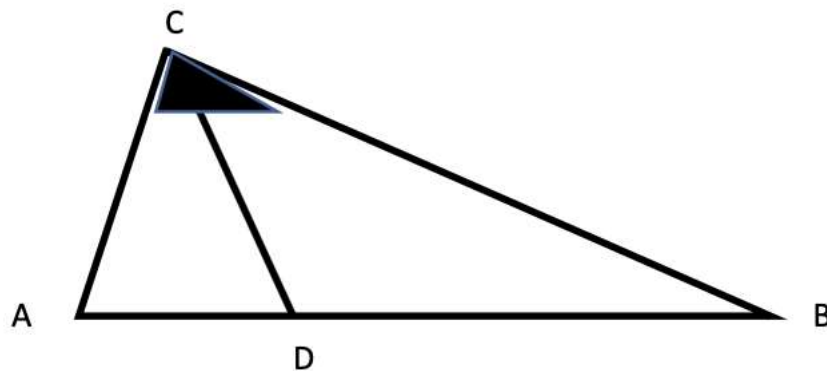(Hint : $K^4 + K^2 + 1 = (K^2 - K + 1)(K^2 + K + 1)$).

[10 marks]

**Question 5**

Let $A$ and $B$ be real matrices of size $m \times n$ and $n \times m$, respectively. Prove that the non-zero eigenvalues of $AB$ and $BA$ are the same.

(Hint : $det(I_m + AB) = det(I_n + BA)$).

[10 marks]

**Question 6**

Given the figure below. Suppose $AC$ is not congruent to $BC$ and $CD$ bisects $\angle ACB$. Prove that $CD$ cannot be perpendicular to $AB$.



[10 marks]